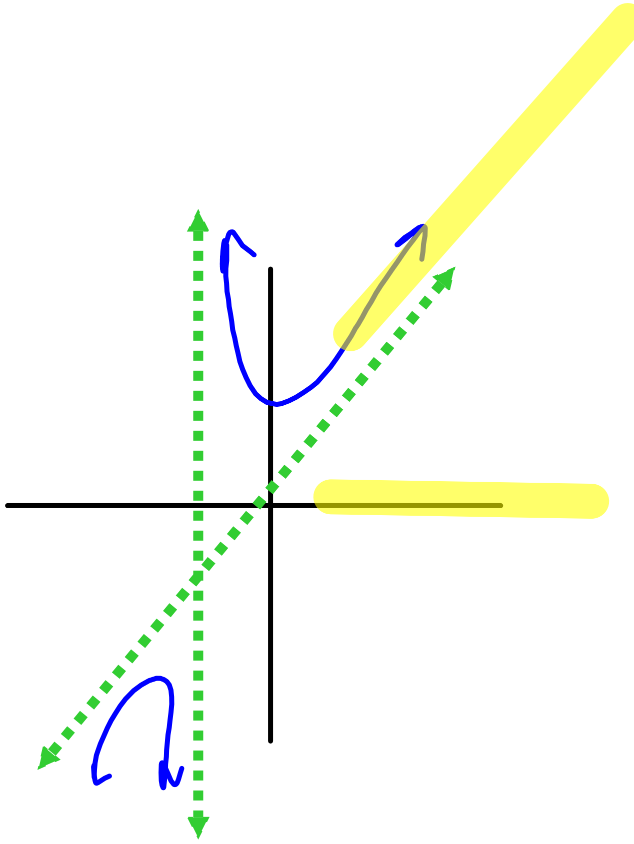
Lesson 11.4
Limits at Infinity

$$f(x) = \frac{2x^3 + x^2 - 8x - 4}{x^2 - 3x + 2}$$



Finding Limits as X approaches "infinity"

$$\lim_{x \to \infty} \frac{1}{x} = 0$$

*Think about this...as x gets larger and larger, what happens?

Limits at infinity:

If r is a positive real number, then

$$\lim_{x\to\infty}\frac{1}{x^r}=0$$

Furthermore, if x^r is defined when x<0, then

$$\lim_{x\to-\infty}\frac{1}{x^r}=0$$

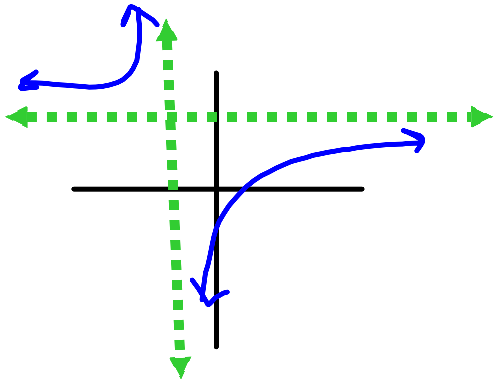
EX: Find the limit

$$\lim_{x\to\infty}\left(4-\frac{3}{x^2}\right) = 4$$

$$\underbrace{\lim_{x\to\infty} 4}_{4} - \underbrace{\lim_{x\to\infty} \frac{3}{x^2}}_{0}$$

Find the limit. Use a graphing calculator to check your work.

$$\lim_{x\to\infty} \frac{5x-2}{2x+1} = \frac{5}{2}$$



*We can also divide everything by the highest degree variable that occurs in the denominator.

$$\lim_{x \to -\infty} \frac{3x^2 - 2x}{2x^2 + 5} = \frac{3}{2}$$

$$\lim_{x\to\infty} \frac{-3x^{100} + 4x^{12}}{4x^{100} + 5x^{9}} = -\frac{3}{4}$$

If the power is the same in the top and the bottom, then...

$$\lim_{x \to -\infty} \frac{-12x^{33} + 4x^{4}}{x^{33} + 5x^{9}} = -12$$

$$\lim_{x\to\infty} \frac{3x + 2}{4x^2 - 1} = 0$$

$$\lim_{x \to -\infty} \frac{3}{2x^3 + 4} = 0$$

If the power is higher in the bottom than the top, then...

$$\lim_{x\to-\infty} \frac{-534x^{32452436} - 2x^{345}}{153x^{344} - 3x^{32452437}} = 0$$

$$\lim_{x \to \infty} \frac{3x^2 + 4}{4x - 1} = \quad \text{DNE}$$

$$\lim_{x \to -\infty} \frac{3x^5 + 2x}{4x^2 - 3x^3} = \text{DNE}$$

If the power is higher in the top than the bottom, then...

$$\lim_{x\to\infty} \frac{3x^{10000} - 2x^{23415}}{1232153x^{2345} - 3x^{3}} =$$ DNE

$$\lim_{x \to -\infty} \frac{-2x^3 + x}{-5x^4 - 1} = \quad 0$$

Let's Recap

$$f(x) = \frac{a_n x^n}{b_m x^m}$$

top < bottom

$$\lim_{x \to \pm\infty} f(x) = \begin{cases} 0 & if\ n < m \\ \dfrac{a_n}{b_m} & if\ n = m \\ DNE & if\ n > m \end{cases}$$

divide coefficients

slant or ∅

$$\lim_{x \to \pm\infty} f(x) = \begin{cases} 0 \quad if\ n < m \\ \dfrac{a_n}{b_m} \quad if\ n = m \\ DNE \quad if\ n > m \end{cases}$$

This should look slightly familiar. We used this earlier this year when we graphed rational functions. This helped us identify our **Horizontal Asymptotes!**

ex: What are the horizontal asymptotes for the following function:

$$f(x) = \frac{2x+1}{5x}$$

Limits of Sequences

Limits of sequences have many of the same properties as limits of functions.

Ex: $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots \frac{1}{32}, \frac{1}{64}, \ldots$

$\lim_{n \to \infty} 2^n$

$$\lim_{n \to \infty} \frac{1}{2^n} = 0$$

has a limit

DNE

The sequence is said to **converge**.

Diverge

A sequence that does not converge is said to **diverge**. For instance, the sequence 1, 3, 5, 7, 9,.... because it does not approach a unique number.

EX: Find the limit of the sequence: $a_n = \dfrac{2n^2+1}{4n^2}$

$$\lim_{n\to} \frac{2n^2+1}{4n^2} = \frac{1}{2}$$